InkMaths

ANSWER BOOK

Algorithms, sorting and bin packing

Decision Mathematics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 A finite sequence of precise instructions that solves a problem and always terminates. [2]
Its order says how the number of operations grows with the size n of the problem: an order n^2 algorithm takes four times as long on a list twice as long.

A2 Pass 1: 5, 3, 9, 1, 12. Pass 2: 3, 5, 1, 9, 12. Pass 3: [3]
3, 1, 5, 9, 12. Pass 4: 1, 3, 5, 9, 12. A fifth pass makes no swaps, which is how the algorithm knows it has finished.

SECTION B · Standard

B1 Pivot 3: gives 1 | 3 | 5, 12, 9. Pivot of 5, 12, 9 is 12: gives 5, 9 | [4]
12 | nothing. Pivot of 5, 9 is 9: gives 5 | 9. The sublist 1 is already sorted. Final order 1, 3, 5, 9, 12, in three pivot steps against bubble sort's four passes.

B2 The total is 45, so the lower bound is 45/10 = 4.5, rounded up to 5 [4]
bins. First fit gives (3, 6, 1), (4, 5), (8, 2), (7), (9): 5 bins. That matches the bound, so this packing is optimal and no further work is needed.

B3 Placing the large items first leaves the small ones to fill the gaps they create, whereas [3]
first fit can commit a bin to small items and then have no room for a large one. Neither looks ahead, so both can be beaten by a packing that sacrifices an early fit for a better later one; finding the optimum in general is a much harder problem.

SECTION C · Stretch

C1 First fit: 6 opens bin 1, the 1 joins it, the second 6 opens bin 2 and its 1 joins bin 1, and so [4]
on. The three 6s and three 1s end as (6, 1, 1, 1), (6), (6), and each 7 then needs its own bin: 6 bins. The total is 42, so the lower bound is 5. First fit decreasing also gives six here, so five may not be achievable at all: falling short of the bound proves nothing on its own.