inkMaths

ANSWER BOOK

Allocation and the Hungarian algorithm

Decision Mathematics 2 · Year FM

6 questions · 23 marks

NAVY EDITION

SECTION A · Warm-up

A1 Every complete allocation uses exactly one cell from each row. Subtracting the same [3] amount from a whole row therefore reduces the total of every allocation by that same amount, so their order is unchanged and the cheapest one before the subtraction is still the cheapest after it. The same argument applies to columns, since every allocation uses one cell from each column too.

A2 Four is fewer than five, so no allocation using only zeros exists yet. Find the smallest [2] uncovered entry, subtract it from every uncovered element, and add it to every element covered by two lines. Then count the minimum covering lines again.

SECTION B · Standard

B1 Row minima are 9, 8, 9, giving rows 5, 0, 3; 3, 0, 7; 4, 1, 0.
Column minima are then 3, 0, [5] 0, giving 2, 0, 3; 0, 0, 7; 1, 1, 0.
The zeros need 3 lines to cover, matching the size of the matrix, so an allocation exists: P to 2, Q to 1, R to 3.
Cost = 9 + 11 + 9 = 29, from the original table.

B2 The largest entry is 15, so subtract every entry from 15, giving 1, 6, 3; 4, [4] 7, 0; 2, 5, 6. Minimise that in the ordinary way.
The best allocation is P to 1, Q to 3, R to 2, with profit 14 + 15 + 10 = 39.
As a check, the reduced total is 1 + 0 + 5 = 6 and 3(15) - 39 = 6, as it must be.

B3 Add a dummy job, a fourth column of zeros, so the matrix is square. [3] Whoever is allocated the dummy is the worker left idle, at no cost.
For the refusal, give that one cell a cost large enough that the algorithm will never choose it, and say in your working that you have done so. The rest of the method runs unchanged.

SECTION C · Stretch

C1 Row minima 6, 9, 5 give 0, 5, 5; 0, 8, 4; 7, 2, 0. Column minima 0, 2, 0 then give 0, [6] 3, 5; 0, 6, 4; 7, 0, 0.
Every zero is covered by column 1 together with row 3, so only 2 lines are needed against a size of 3, and the test fails.
The uncovered entries are 3, 5, 6 and 4, so the smallest is 3. Subtract 3 from each of those and add 3 to the doubly covered entry, giving 0, 0, 2; 0, 3, 1; 10, 0, 0.
Three lines are now needed, so an allocation exists: P to 2, Q to 1, R to 3, costing 11 + 9 + 5 = 25. Checking all six allocations confirms 25 is the least.