inkMaths

ANSWER BOOK

Areas with polar coordinates

Polar coordinates · Year FM

6 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $A = \frac{1}{2}\int r^2\, d\theta$ from α to β. Each thin wedge is nearly a triangle with base $r\, d\theta$ and height r, and a triangle's half shows up in front of the $r^2\, d\theta$. [2]

A2 $A = \frac{1}{2}\int 16\, d\theta = 8 \times \pi/3 = 8\pi/3$. Check against geometry: the full circle has area 16π, and a sixth of it is $8\pi/3$, matching the sixth of a turn. [3]

SECTION B · Standard

B1 $A = \frac{1}{2}\int 4(1 + \cos\theta)^2\, d\theta$ over 0 to 2π $= 2\int(1 + 2\cos\theta + \cos^2\theta)\, d\theta$. The $\cos\theta$ term dies over a full turn and $\cos^2\theta$ contributes π, so $A = 2(2\pi + \pi) = 6\pi$. Squaring first and reaching for the double angle is the whole method. [4]

B2 $A = \frac{1}{2}\int 9\cos^2 2\theta\, d\theta = (9/4)\int(1 + \cos 4\theta)\, d\theta$ over the petal. The $\cos 4\theta$ part vanishes across the symmetric limits, leaving $(9/4)(\pi/2) = 9\pi/8$. The limits are where $r = 0$: consecutive zeros of $\cos 2\theta$ bound the loop. [4]

B3 $A = \frac{1}{2}\int 36\sin^2\theta\, d\theta = 9\int(1 - \cos 2\theta)\, d\theta$ over 0 to π $= 9\pi$, which is $\pi \times 3^2$, the circle's area. The circle is traced once by θ from 0 to π; running to 2π would count it twice. [3]

SECTION C · Stretch

C1 Between $\pi/4$ and $\pi/2$, $\cos 2\theta$ is negative: the curve is tracing a different petal on the opposite side of the pole, not the loop in question. Squaring hides the sign, so the integral silently adds a second half-petal's worth of area. Limits must run between consecutive zeros of r, here $-\pi/4$ to $\pi/4$. [4]