inkMaths

ANSWER BOOK

Arithmetic series

Sequences and series · Year 12–13

7 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $u_{15} = 7 + 14 \times 5 = 77$. The 15th term has taken fourteen steps, not fifteen: the bracket in $a + (n - 1)d$ is where this topic catches people. [2]

A2 $a = 4$ and $d = 7$, so $u_n = 4 + 7(n - 1) = 7n - 3$. Setting $7n - 3 = 200$: $n = 29$, a whole number, so 200 is the 29th term. A fractional n would have meant no. [2]

SECTION B · Standard

B1 From 4th to 9th is five steps: $5d = 38 - 18 = 20$, so $d = 4$. Then $a + 3d = 18$ gives $a = 6$, and $u_{25} = 6 + 24 \times 4 = 102$. Differences of terms count steps between them, which beats setting up simultaneous equations from scratch. [3]

B2 $a = 8$ and $d = 3$, so $S_{30} = 15 \times (16 + 29 \times 3) = 15 \times 103 = 1545$. The last-term form agrees: $l = 8 + 29 \times 3 = 95$, and $15 \times (8 + 95) = 1545$. Two dressings of one formula make a built-in check. [3]

B3 Write $S = 1 + 2 + \ldots + n$ forwards and backwards and add the two lines: each column pairs to $n + 1$, and there are n columns, so $2S = n(n + 1)$ and $S = n(n + 1)/2$. For $n = 200$: $200 \times 201/2 = 20\,100$. The pairing argument is the required proof, and it is two lines long. [4]

B4 The deposits are arithmetic with $a = 20$ and $d = 4$, so $S_{24} = 12 \times (40 + 23 \times 4) = 12 \times 132 =$ £1584. The model's gradient is the £4 escalation; the question is a sum, not a term, because every month's deposit stays in the pot. [3]

SECTION C · Stretch

C1 $S_n = n(2 \times 5 + 4(n - 1))/2 = n(2n + 3)$ must exceed 1000. Trying the boundary: $S_{21} = 945$, too small, and $S_{22} = 1034$, past the mark. So 22 terms are needed. Solving the quadratic $2n^2 + 3n - 1000$ > 0 gives n > 21.6, the same verdict; either route must end with whole terms, rounded up. [3]