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ANSWER BOOK

Calculus with hyperbolic functions

Hyperbolic functions · Year FM

6 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 sinh' = cosh and cosh' = sinh: the pair swap with both signs positive. Unlike cos, whose derivative is -sin, cosh differentiates with no minus anywhere. [2]

A2 Chain rule: $dy/dx = 3 \sinh 3x$. At $x = 0$, $\sinh 0 = 0$, so the gradient is 0: the curve has its minimum there, value $\cosh 0 = 1$. [3]

SECTION B · Standard

B1 Gradient $\sinh(\ln 2) = 3/4$ and value $\cosh(\ln 2) = 5/4$, so $y = 5/4 + (3/4)(x - \ln 2)$. Exact hyperbolic values at logarithmic points come straight from the definitions, no calculator required. [4]

B2 $\int \sinh = \cosh$, both signs positive: $[\cosh x]$ from 0 to $\ln 3 = \cosh(\ln 3) - 1 = 5/3 - 1 = 2/3$. The value $\cosh(\ln 3) = 5/3$ comes from $(3 + 1/3)/2$. [3]

B3 $a = 3$: the integral is $\operatorname{arcosh}(x/3)$ from 3 to 5 $= \operatorname{arcosh}(5/3) - \operatorname{arcosh} 1$. In log form: $\ln(5/3 + \sqrt{(25/9 - 1)}) = \ln(5/3 + 4/3) = \ln 3$, and $\operatorname{arcosh} 1 = 0$. The answer is $\ln 3$. [3]

SECTION C · Stretch

C1 $\cosh^2 x = (1 + \cosh 2x)/2$, so the integral is $[x/2 + \sinh 2x/4]$ from 0 to $1 = 1/2 + \sinh 2/4$. Since $\sinh 2 = 2 \sinh 1 \cosh 1$, this is $(1 + \sinh 1 \cosh 1)/2 \approx 1.407$. The double angle identity carries no sign flip, unlike its cos 2x counterpart. [4]

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