inkMaths

ANSWER BOOK

Calculus with inverse trigonometric functions

Further calculus · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $d/dx\,(\arcsin x) = 1/\sqrt{1 - x^2}$ and $d/dx\,(\arctan x) = 1/(1 + x^2)$. Both are algebraic: the trig disappears entirely when the implicit differentiation is carried out. [2]

A2 Differentiate implicitly: $\sec^2 y\,(dy/dx) = 1$. Since $\sec^2 y = 1 + \tan^2 y = 1 + x^2$, $dy/dx = 1/(1 + x^2)$. The identity converts the trig in the denominator into algebra in x. [3]

SECTION B · Standard

B1 The chain rule doubles the numerator: $dy/dx = 2/\sqrt{1 - 4x^2}$. At $x = 1/4$: $2/\sqrt{1 - 1/4} = 2/\sqrt{3/4} = 4/\sqrt{3} \approx 2.31$. The inner derivative 2 is the piece most commonly dropped. [3]

B2 $a = 2$, so the integral is $(1/2)\arctan(x/2)$, evaluated from 0 to 2: $(1/2)(\arctan 1 - \arctan 0) = (1/2)(\pi/4) = \pi/8$. The arctan pattern carries the $1/a$ factor; forgetting it halves nothing and doubles the answer. [4]

B3 $a = 3$: the integral is $\arcsin(x/3)$ with no front factor. At the limits: $\arcsin(1/2) - \arcsin 0 = \pi/6$. Exact multiples of π drop out whenever the upper limit lands on a friendly value of sine. [4]

SECTION C · Stretch

C1 $x^2 + 2x + 5 = (x + 1)^2 + 4$, so the integral is $(1/2)\arctan((x + 1)/2)$. From -1 to 1: $(1/2)(\arctan 1 - \arctan 0) = \pi/8$. Completing the square converts any irreducible quadratic denominator into the $a^2 + u^2$ pattern with $u = x + 1$. [4]