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ANSWER BOOK

Centre of mass of a discrete distribution

Further Mechanics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 [2]

Taking moments about the origin, each particle contributes its mass times its distance, and the total must equal the whole mass placed at the centre of mass. The masses are therefore the weights, so the point is pulled towards the heavier particles. Only when all the masses are equal does it reduce to the plain average.

A2 [3]

$\Sigma mx = 3(0) + 5(4) + 2(10) = 0 + 20 + 20 = 40$, and the total mass is 10 kg. So $x(G) = 40/10$ = 4, which lies between the outer masses as it must.

SECTION B · Standard

B1 [4]

Total mass 10 kg. $\Sigma mx = 0 + 4 + 6 + 0 = 10$, so $x(G)$ = 1. $\Sigma my = 0 + 0 + 9 + 12 = 21$, so $y(G)$ = 2.1. The point (1, 2.1) sits towards the top of the rectangle, pulled there by the two heavier masses at $y = 3$.

B2 [4]

The formula gives $(4 \times 2 + 8m)/(4 + m) = 5$, so $8 + 8m = 20 + 5m$ and $3m = 12$, giving m = 4 kg. Checking: $(8 + 32)/8 = 5$. The centre is halfway between them, which is what equal masses always produce.

B3 [3]

Supported at that point it balances, because the moments of the weights about it cancel. And suspended freely from any point, it hangs with the centre of mass vertically below that point, since otherwise the weight would have a moment about the pivot and the body would turn.

SECTION C · Stretch

C1 [4]

The new total mass is 15 kg. For x: $15(2) = 10 + 5a$, so $5a = 20$ and a = 4. For y: $15(2) = 21 + 5b$, so $5b = 9$ and b = 1.8. The mass goes at (4, 1.8), outside the original rectangle, which is what it takes to drag the centre that far right.