inkMaths

ANSWER BOOK

Centres of mass of plane figures and frameworks

Further Mechanics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

© 2026 InkMaths. All rights reserved.
inkmaths.co.uk

SECTION A · Warm-up

A1 A rectangle's centre of mass is at its centre, where the diagonals cross. A triangle's is at the centroid, a third of the way from each side towards the opposite vertex. Any axis of symmetry must contain the centre of mass, so a symmetric shape needs only one coordinate calculating. [1]

A2 Areas 12 and 8, total 20, with centres (3, 1) and (1, 4). $x(G) = (12 \times 3 + 8 \times 1)/20 = 44/20 =$ 2.2. $y(G) = (12 \times 1 + 8 \times 4)/20 = 44/20 =$ 2.2. The L is symmetric about the line $y = x$, which is why the two coordinates agree. [3]

SECTION B · Standard

B1 Areas 16π and 4π, leaving 12π. By symmetry the centre of mass lies on the x-axis. Treating the hole as a negative area: $x(G) = (16\pi \times 0 - 4\pi \times 2)/(12\pi) = -8/12 =$ -2/3. It has moved away from the hole, as it must, and the π has cancelled throughout. [4]

B2 Lengths 6, 4 and 4, total 14, acting at (3, 0), (6, 2) and (0, 2). $x(G) = (18 + 24 + 0)/14 = 42/14 =$ 3, which symmetry about $x = 3$ confirms. $y(G) = (0 + 8 + 8)/14 = 16/14 =$ 8/7 ≈ 1.14, below the middle because the long rod lies along the bottom. [4]

B3 The lamina has its mass spread over the area, so each element is weighted by area and the answer is the centroid of the region. The framework has its mass along the perimeter, so each rod is weighted by its length and acts at its own midpoint. Different distributions of the same total mass give different balance points. [3]

SECTION C · Stretch

C1 The centroid of a triangle is the average of its vertices, so $x(G) = (0 + 4 + 0)/3 =$ 4/3 and $y(G) = (0 + 0 + 3)/3 =$ 1. The framework's is at (1.5, 1), so the y coordinates happen to agree while the x coordinates do not. The framework sits further right because the hypotenuse, its longest rod, has its midpoint at $x = 2$. [4]

© 2026 InkMaths. All rights reserved.