inkMaths

ANSWER BOOK

Combinations of normal random variables

Further Statistics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 It is normal, with mean $a\mu_x + b\mu_y$ and variance $a^2\sigma_x^2 + b^2\sigma_y^2$. The variables must be independent; without that the variances cannot simply be added. [2]

A2 X + Y is N(19, 25). X - Y is N(5, 25). The means follow the sign but the variances add in both cases, so both have standard deviation 5. [3]

SECTION B · Standard

B1 P(X < Y) = P(X - Y < 0), and X - Y is N(5, 25) with standard deviation 5. Standardising: z = (0 - 5)/5 = -1, so the probability is P(Z < -1) = 0.1587. Using variance 9 - 16 would give a negative variance, which is the check that the rule must be addition. [4]

B2 Five independent readings: mean 60, variance 5 × 9 = 45, so N(60, 45) with standard deviation 6.71. Five times one reading: mean 60, variance 5^2 × 9 = 225, so N(60, 225) with standard deviation 15. The five separate readings vary independently and their errors partly cancel; scaling one reading magnifies its error fivefold. [4]

B3 Mean = 2(12) - 3(7) = 24 - 21 = 3. Variance = 2^2(9) + 3^2(16) = 36 + 144 = 180. So 2X - 3Y is N(3, 180), with standard deviation 13.4. Both coefficients square, including the one attached to the subtraction. [3]

SECTION C · Stretch

C1 The total for eight people is normal with mean 560 and variance 8 × 100 = 800, so the standard deviation is √800 = 28.28. Standardising: z = (600 - 560)/28.28 = 1.414, so P(total > 600) = 1 - 0.9214 = 0.0786, about one time in thirteen. The independence assumption is doing real work here: a group travelling together may well not be independent. [4]