inkMaths

ANSWER BOOK

Combinatorics

Further Pure 2 · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

A1 Ordered: $10 \times 9 \times 8 \times 7$ = 5040, which is ${}^{10}P_4$. Unordered: divide by the 4! = 24 orderings of each selection, giving 210 = ${}^{10}C_4$. The two always differ by exactly r!. [3]

A2 Fix one person to remove the rotational freedom, then arrange the other 7 in the remaining seats: 7! = 5040 ways. Counting 8! would treat each arrangement as 8 different ones, once for each rotation. [3]

SECTION B · Standard

B1 There are 10 letters, with S appearing 3 times, T 3 times and I twice; A and C appear once each. Dividing the 10! orderings by the indistinguishable rearrangements gives 10!/(3! 3! 2!) = 3628800/72 = 50400. [4]

B2 Total committees: ${}^{12}C_5$ = 792. Count the complement instead. No women: ${}^{7}C_5$ = 21. Exactly one woman: $5 \times {}^{7}C_4 = 5 \times 35$ = 175. So the answer is 792 − 21 − 175 = 596. Adding the cases with 2, 3, 4 and 5 women gives the same total in four times the work. [4]

B3 Multiples of 3: 166. Multiples of 5: 100. Multiples of 15, counted in both: 33. By inclusion and exclusion the answer is 166 + 100 − 33 = 233. Forgetting the overlap gives 266 and counts every multiple of 15 twice. [3]

SECTION C · Stretch

C1 The total number of subsets is 2^{10} = 1024, since each element is independently in or out. Expanding $(1 - 1)^{10}$ by the binomial theorem gives the alternating sum of the ${}^{10}C_r$, which is 0, so the even-sized and odd-sized subsets are equal in number. Each group therefore has 1024/2 = 512 members. [4]

