inkMaths

ANSWER BOOK

Comparing two normal means

Further Statistics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Each mean has variance σ^2/n: $50/25 = 2$ and $30/25 = 1.2$. Adding gives 3.2, so the standard error is $\sqrt{3.2}$ = 1.789 to three decimal places. The variances are added, never subtracted. [2]

A2 Standard error = $\sqrt{(9/36 + 16/49)} = \sqrt{(0.25 + 0.3265)} = \sqrt{0.5765}$ = 0.759. The difference of means is 1.8, so $z = 1.8/0.759$ = 2.37. [3]

SECTION B · Standard

B1 H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$, two-tailed at 5%, so the critical values are ±1.96. The statistic 2.37 exceeds 1.96, so it lies in the critical region: reject H_0. There is evidence at the 5% level that the two population means differ, with the first appearing larger. [4]

B2 The point estimate is 1.8 and the standard error 0.759, so the interval is 1.8 ± 1.96 × 0.759 = 1.8 ± 1.488, that is (0.31, 3.29). Zero lies outside, which is exactly the verdict the two-tailed test at 5% reached. The interval says more: the difference could plausibly be anywhere from a third of a unit to over three. [4]

B3 Replace each σ^2 by the sample variance s^2 from that sample. The Central Limit Theorem keeps the sample means approximately normal whatever the population shape, and with large samples the estimated variances are close enough to the real ones for the normal critical values to remain usable. The conclusion becomes approximate rather than exact, and this should be stated. [3]

SECTION C · Stretch

C1 H_0: $\mu_1 = \mu_2$; H_1: μ_1 > μ_2. The statistic is 2.371, which exceeds 2.3263, so reject H_0 at the 1% level as well. The margin is small: had the difference been 1.77 rather than 1.8 the 1% test would have failed while the 5% test still passed. A result close to a critical value should be reported as such rather than as a firm verdict. [4]