inkMaths

ANSWER BOOK

Complex arithmetic and the Argand diagram

Complex numbers · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $z + w = 4 - 2i$ and $z - w = 2 + 6i$. Real parts combine with real, imaginary with imaginary; nothing crosses over. [2]

A2 $zw = 3 - 12i + 2i - 8i^2 = 3 - 10i + 8 = 11 - 10i$. The whole calculation is expanding brackets, with $i^2 = -1$ the only new step, converting the $-8i^2$ into $+8$. [3]

SECTION B · Standard

B1 Multiply top and bottom by the conjugate $1 + i$: the denominator becomes $(1 - i)(1 + i) = 2$ and the numerator $(2 + 3i)(1 + i) = 2 + 2i + 3i - 3 = -1 + 5i$. So the quotient is $-1/2 + (5/2)i$. Realising the denominator is the entire method. [4]

B2 $|z| = \sqrt{9 + 16} = 5$. z sits at the point $(3, 4)$ in the first quadrant; the conjugate $3 - 4i$ sits at $(3, -4)$. Conjugation is reflection in the real axis, and both points are distance 5 from the origin. [3]

B3 The discriminant is $16 - 52 = -36$, so $z = (4 \pm 6i)/2 = 2 \pm 3i$. The roots are a conjugate pair, as the roots of any quadratic with real coefficients must be when they are not real: the $\pm$ of the quadratic formula lands on $\pm 6i$. [4]

SECTION C · Stretch

C1 Expand: $(2x + y) + (2y - x)i = 5$. Matching parts: $2x + y = 5$ and $2y - x = 0$, so $x = 2y$, giving $5y = 5$, $y = 1$ and $x = 2$. So $x + yi = 2 + i$. Check: $(2 + i)(2 - i) = 4 + 1 = 5$. Equating real and imaginary parts turns one complex equation into two real ones. [4]