



ANSWER BOOK

Compound angles and the harmonic form

Trigonometry · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $\cos 75^\circ = \cos (45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ = (\sqrt{2}/2)(\sqrt{3}/2) - (\sqrt{2}/2)(1/2)$
 $= (\sqrt{6} - \sqrt{2})/4$. Cosine's middle sign flips: the plus inside the bracket becomes a minus outside, the detail most often mis-copied.

A2 $\cos A = 4/5$ from the 3-4-5 triangle, and $\sin 2A = 2 \sin A \cos A = 2 \times (3/5)(4/5) = 24/25$.
 Doubling the angle is not doubling the sine: $\sin 2A$ needs both ratios.

SECTION B · Standard

B1 The cosine-only dressing of the double angle formula does it in one step: $\cos 2A = 2 \cos^2 A - 1 = 2/9 - 1 = -7/9$. No need to find $\sin A$, and no need to know which quadrant A is in: squaring erases the question.

B2 $R = \sqrt{(25 + 144)} = 13$ and $\tan \alpha = 12/5$ gives $\alpha = 1.176$. So $5 \sin \theta + 12 \cos \theta = 13 \sin (\theta + 1.176)$. The maximum is 13, where the sine reaches 1: $\theta + 1.176 = \pi/2$, so $\theta = 0.395$. Two waves in, one wave out, and the maximum read off with no calculus.

B3 Expand the double angle: $2 \sin x \cos x = \sin 2x$, gather and factorise: $\sin x (2 \cos x - 1) = 0$.
 $\sin x = 0$ gives $x = 0^\circ$ and 180° ; $\cos x = 1/2$ gives $x = 60^\circ$ and 300° . Four solutions: $0^\circ, 60^\circ, 180^\circ, 300^\circ$. Dividing by $\sin x$ at the start would have silently discarded half of them.

SECTION C · Stretch

C1 Expanding $R \cos (\theta + \alpha) = R \cos \alpha \cos \theta - R \sin \alpha \sin \theta$ and matching: $R \cos \alpha = 3$ and $R \sin \alpha = 4$, so $R = \sqrt{(9 + 16)} = 5$ and $\tan \alpha = 4/3$, giving $\alpha = 0.927$. The expression is $5 \cos (\theta + 0.927)$, with minimum -5 where the cosine bottoms out at -1.

C2 Expanding both compound angles: $(\sin A \cos B + \cos A \sin B) + (\sin A \cos B - \cos A \sin B) = 2 \sin A \cos B$, the cross terms cancelling. With $A = 45^\circ$ and $B = 30^\circ$: $\sin 75^\circ + \sin 15^\circ = 2 \sin 45^\circ \cos 30^\circ = 2 \times (\sqrt{2}/2)(\sqrt{3}/2) = \sqrt{6}/2$. Two awkward sines collapse into one clean surd.