inkMaths

ANSWER BOOK

Conditional probability

Statistics · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $P(A|B) = 0.18/0.6 = 0.3$: the overlap as a share of B's world. Independence means conditioning changes nothing, so $P(A)$ would have to equal 0.3 as well. [2]

SECTION B · Standard

B1 Two routes end in a blue second draw: red then blue, $4/7 \times 3/6 = 12/42$, and blue then blue, $3/7 \times 2/6 = 6/42$. Adding: $18/42 = 3/7$. Both red is one route: $4/7 \times 3/6 = 2/7$. The second branch's probabilities shift because the bag has changed: that shift is the whole meaning of 'without replacement'. [4]

B2 $P(\text{first red} \mid \text{second blue}) = P(\text{red then blue})/P(\text{second blue}) = (12/42)/(18/42) = 2/3$. Conditioning on the later event to ask about the earlier one feels backwards, but the formula has no sense of time: it only compares an overlap with a whole. [4]

B3 From the table rows: $P(\text{attends} \mid \text{adult}) = 45/60 = 3/4$. From the attends column, which holds 55 members: $P(\text{adult} \mid \text{attends}) = 45/55 = 9/11$. Same overlap, two different denominators: the conditioning event decides which world the fraction lives in. [4]

SECTION C · Stretch

C1 A and B are independent exactly when $P(A|B) = P(A)$: learning that B happened leaves A's probability untouched. That is the everyday meaning of 'no influence', and dividing $P(A \cap B) = P(A)P(B)$ by $P(B)$ shows the two definitions are one. [3]

C2 The patient has swapped $P(\text{positive} \mid \text{condition})$ for $P(\text{condition} \mid \text{positive})$, and the two have different denominators. When the condition is rare, the positives are dominated by false alarms from the huge healthy group, so $P(\text{condition} \mid \text{positive})$ can be far below 99%. The direction of the bar is everything. [3]