



ANSWER BOOK

Confidence intervals and tests with the t-distribution

Further Statistics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** It is used when the population is normal but its variance is unknown and estimated by s^2 from the same sample, particularly when that sample is small. The t-distribution is symmetric about zero like the normal but has heavier tails, so its critical values are further out, and it approaches the normal as the degrees of freedom grow.
- A2** $H_0: \mu = 50$; $H_1: \mu > 50$, with 15 degrees of freedom. Standard error = $6/\sqrt{16} = 1.5$, so $t = (52.5 - 50)/1.5 = 1.667$. Since $1.667 < 1.753$, do not reject H_0 : there is insufficient evidence at the 5% level that the mean exceeds 50.

SECTION B · Standard

- B1** The interval is $52.5 \pm 2.131 \times 1.5 = 52.5 \pm 3.20$, that is $(49.30, 55.70)$. The value 50 lies inside, which agrees with not rejecting H_0 . Using 1.96 instead of 2.131 would have given $(49.56, 55.44)$, too narrow: the estimated variance has to be paid for.
- B2** $H_0: \mu_d = 0$; $H_1: \mu_d > 0$, with 7 degrees of freedom. The mean difference is $16/8 = 2$, and the squared deviations sum to 28, so $s^2 = 28/7 = 4$ and $s = 2$. Standard error = $2/\sqrt{8} = 0.707$, so $t = 2/0.707 = 2.83$. Since $2.83 > 1.895$, reject H_0 : there is evidence at the 5% level that the treatment raises the measurement.
- B3** Subtracting within pairs removes the variation between subjects, which is usually far larger than the effect being measured. What is left is only the variation in the effect itself, so the standard error is much smaller and a genuine difference is easier to detect. Treating paired data as two independent samples discards that advantage.

SECTION C · Stretch

- C1** Pooled variance = $(5 \times 5.76 + 8 \times 9.61)/13 = 105.68/13 = 8.129$, so $s = 2.851$. Standard error = $2.851 \times \sqrt{(1/6 + 1/9)} = 2.851 \times 0.527 = 1.503$. Then $t = 2.4/1.503 = 1.60$ on 13 degrees of freedom. Since $1.60 < 2.160$, do not reject H_0 : insufficient evidence at the 5% level of a difference. The assumption of equal variances would itself be worth checking with an F test.