inkMaths

ANSWER BOOK

Conic sections

Further Pure 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Parabola: $(at^2, 2at)$. Rectangular hyperbola: $(ct, c/t)$. Both turn two-variable problems into one-variable algebra, which is why examiners lead with them. [2]

A2 $4a = 20$, so $a = 5$. The focus is $(5, 0)$ and the directrix is $x = -5$. Quartering rather than halving the coefficient is the step most often slipped. [3]

SECTION B · Standard

B1 $a = 4$ and $b^2 = 7$. From $b^2 = a^2(1 - e^2)$: $7 = 16(1 - e^2)$, so $e^2 = 9/16$ and $e = 3/4$. Foci $(\pm ae, 0) = (\pm 3, 0)$; directrices $x = \pm a/e = \pm 16/3$. The minus sign inside the bracket is what distinguishes the ellipse formula from the hyperbola's. [4]

B2 Here $b^2 = a^2(e^2 - 1)$: $9 = 16(e^2 - 1)$, so $e^2 = 25/16$ and $e = 5/4$. Foci $(\pm 5, 0)$. From $(4, 0)$ the distances are 1 and 9, differing by $8 = 2a$. A hyperbola holds the difference of focal distances constant, as an ellipse holds the sum. [4]

B3 Every point of the conic satisfies distance to the focus = e × distance to the directrix. When e < 1 the curve closes into an ellipse; e = 1 balances it into a parabola; e > 1 opens it into a hyperbola's two branches. One definition with a dial produces all three. [3]

SECTION C · Stretch

C1 $16^2 = 256$ and $16 \times 16 = 256$, so P is on the curve. Here $4a = 16$ so $a = 4$: the focus is $(4, 0)$ and the directrix $x = -4$. Distance from P to the focus: $\sqrt{(12^2 + 16^2)} = 20$. Distance to the directrix: $16 + 4 = 20$. Equal, exactly as $e = 1$ demands. [4]