inkMaths

ANSWER BOOK

Continuous random variables: density and distribution functions

Further Statistics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Probability is the area under the density, and the area over a single point is zero, so no individual value carries any probability. It follows that P(X < a) and P(X $\leq$ a) are equal, so the inequality signs make no difference: a genuine simplification compared with discrete distributions. [2]

A2 The integral of 4 - x^2 from 0 to 2 is [4x - x^3/3] = 8 - 8/3 = 16/3. Setting k × 16/3 = 1 gives k = 3/16. The density is non-negative throughout the interval, so it is valid. [3]

SECTION B · Standard

B1 F(x) = (3/16)(4x - x^3/3) = 3x/4 - x^3/16 for 0 $\leq$ x $\leq$ 2, with F = 0 below 0 and F = 1 above 2. Checking: F(2) = 1.5 - 0.5 = 1. Then P(X < 1) = F(1) = 0.75 - 0.0625 = 0.6875. [4]

B2 F(x) = x^2/16 on the interval. The median solves m^2/16 = 0.5, so m^2 = 8 and m = 2√2 = 2.828. The lower quartile solves q^2/16 = 0.25, so q^2 = 4 and q = 2. Both lie inside the range, as they must. [4]

B3 Differentiating, f(x) = x/8 on the interval and zero elsewhere. P(1 < X < 3) = F(3) - F(1) = 9/16 - 1/16 = 8/16 = 0.5. Subtracting values of F is quicker than integrating. [3]

SECTION C · Stretch

C1 The two triangles each have area 1/2, so the total is 1, and f is non-negative throughout. For 0 $\leq$ x < 1, F(x) = x^2/2. For 1 $\leq$ x $\leq$ 2, F(x) = 1/2 + [2t - t^2/2] from 1 to x = 2x - x^2/2 - 1, and F(2) = 4 - 2 - 1 = 1 as required. Then P(X < 1.5) = 3 - 1.125 - 1 = 0.875. The two branches must agree at x = 1, where both give 0.5. [4]