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ANSWER BOOK

Definite integrals and areas

Integration · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Antidifferentiate inside brackets: $[x^4/4]$ from 0 to 2 = 16/4 - 0 = 4. Upper limit first, lower limit subtracted, and the + c never appears: it would cancel in the subtraction. [2]

A2 $[x^2 + x]$ from 1 to 4 = 20 - 2 = 18. A quick geometric check: the region is a trapezium of parallel sides 3 and 9 and width 3, and $(3 + 9)/2 \times 3 = 18$. [2]

SECTION B · Standard

B1 The curve meets the axis at x = 0 and 6, and arches above between them. Area = $\int_0^6 (6x - x^2)\,dx = [3x^2 - x^3/3]$ from 0 to 6 = 108 - 72 = 36. Roots first, then integrate between them. [3]

B2 The curve dips below the axis between its roots 0 and 2, then rises. Integrating piecewise with $F(x) = x^3/3 - x^2$: from 0 to 2 gives -4/3, an area of 4/3 below; from 2 to 3 gives +4/3 above. Total area 8/3. The single integral from 0 to 3 gives 0: the signed pieces cancel, which is exactly why area questions demand the split. [4]

B3 Divide through first: $x^{3/2} + 3x^{-1/2}$. Integrating: $(2/5)x^{5/2} + 6x^{1/2}$. At x = 4: 64/5 + 12 = 124/5; at x = 1: 2/5 + 6 = 32/5. The value is 124/5 - 32/5 = 92/5. The algebra happens before the integral sign ever acts: no rule integrates a quotient directly. [4]

SECTION C · Stretch

C1 $[x^4/4]$ from -2 to 2 = 4 - 4 = 0. The cubic's tail below the axis on the left mirrors its rise above on the right, and the signed areas cancel exactly: odd symmetry doing the arithmetic. [2]

C2 Each half contributes $\int_0^2 x^3\,dx = 4$, one above the axis and one below, so the total area is 8. Zero integral, eight units of area: the definite integral counts signed area, and only the split recovers the geometry. [3]