inkMaths

ANSWER BOOK

# Determinants and inverses

Matrices · Year FM

6 questions · 20 marks

NAVY EDITION

## SECTION A · Warm-up

**A1** det = $3 \times 2 - 1 \times 1 = 5$. Areas are scaled by 5, and the positive sign means orientation is preserved: no reflection is hiding in the transformation. [2]

**A2** Swap the diagonal, negate the off-diagonal, divide by the determinant 5: $(1/5) \times$ rows $(2, -1)$, $(-1, 3)$. Multiplying back gives $(1/5) \times$ rows $(5, 0)$, $(0, 5) = I$, as required. [3]

## SECTION B · Standard

**B1** Expand along the top row: $1 \times (1 \times 1 - 4 \times 0) - 2 \times (3 \times 1 - 4 \times 2) + 0 = 1 - 2 \times (-5) = 11$. The zero in the top row kills a third of the work; expand along whichever row or column carries the most zeros. [4]

**B2** det = $3 \times 2 - 0 \times 1 = 6$, so areas scale by 6: the image has area 24. The determinant is the area scale factor, whatever the shape. [3]

**B3** Singular means det = 0: $k - 6 = 0$, so $k = 6$. The transformation then squashes the plane onto a line, area scale factor zero, and no inverse can exist because the squash cannot be undone. [3]

## SECTION C · Stretch

**C1** Multiply: $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AA^{-1} = I$, and likewise on the other side, so $B^{-1}A^{-1}$ is the inverse of $AB$. The order reverses because undoing a sequence means undoing the last action first, like unwrapping layers in the reverse order they went on. [5]