InkMaths

ANSWER BOOK

Differentiating powers of x

Differentiation · Year 12-13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $dy/dx = 5x^4 - 6x$. The constant 7 contributes nothing: a flat line has gradient zero, wherever it sits. [2]

A2 Rewrite as $x^{1/2}$: the rule gives $(1/2)x^{-1/2}$, which is $1/(2\sqrt{x})$. The rewrite comes first; the power rule cannot see a root sign. [2]

SECTION B · Standard

B1 Rewrite the fraction as $-2x^{-1}$: then $dy/dx = 12x^2 + 2x^{-2} = 12x^2 + 2/x^2$. The minus times the -1 exponent turns positive, the sign slip this question exists to catch. [3]

B2 Rewrite as $6x^{1/2} + 3x^{-2} - 5x$, then apply the rule termwise: $3x^{-1/2} - 6x^{-3} - 5$. In the original notation, $dy/dx = 3/\sqrt{x} - 6/x^3 - 5$. Every term must become a power before the rule touches it. [4]

B3 Expand first: $y = x^5 - 4x^2$, so $dy/dx = 5x^4 - 8x$. The rule handles sums, not products: differentiating each factor and multiplying would give $2x \times 3x^2 = 6x^3$, which is wrong. [3]

B4 Divide through first: $y = 2x - 5/x = 2x - 5x^{-1}$, so $dy/dx = 2 + 5x^{-2} = 2 + 5/x^2$. A quotient with a single-term denominator splits by division; no quotient rule is needed, or yet available. [3]

SECTION C · Stretch

C1 $dy/dx = 3x^2 - 3$, so at $x = 2$ the gradient is 9. Setting $3x^2 - 3 = 0$ gives $x^2 = 1$, so $x = 1$ and $x = -1$: the two flat points of the cubic, a hilltop and a valley the next lesson will learn to tell apart. [3]