inkMaths

ANSWER BOOK

Differentiating trig, exponentials and logs

Differentiation · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 sin 2x gives 2 cos 2x; cos 3x gives −3 sin 3x. The inner coefficient multiplies out front, and cosine's derivative carries the minus sign. [2]

A2 e^{5x} gives $5e^{5x}$: k times itself. ln x gives 1/x, the one derivative on the shelf that leaves its own family entirely. [2]

SECTION B · Standard

B1 Termwise with the shelf: 2 sin 3x gives 6 cos 3x, and 4 cos 2x gives −8 sin 2x. So dy/dx = 6 cos 3x − 8 sin 2x. Each term's inner coefficient multiplies its own front constant. [3]

B2 dy/dx = 5/x + 2 cos 2x. At x = π: 5/π + 2 cos 2π = 5/π + 2 = 3.59. The logarithm's derivative still has an x in it, so the gradient genuinely depends on where it is measured. [3]

B3 The chord gradient is [sin (x + h) − sin x]/h = [sin x cos h + cos x sin h − sin x]/h. Regroup: sin x (cos h − 1)/h + cos x (sin h/h). As h → 0, the small-angle approximations give (cos h − 1)/h → 0 and sin h/h → 1, leaving cos x. The proof is the compound-angle formula and the small angles working together, and it needs radians throughout. [4]

B4 dy/dx = $2e^{2x} + 4e^{-x}$: the −1 in the second exponent meets the −4 in front and turns positive. At x = 0 both exponentials equal 1, so the gradient is 2 + 4 = 6. [3]

SECTION C · Stretch

C1 tan kx differentiates to k $\sec^2$ kx, so dy/dx = 3 $\sec^2$ 3x. At x = 0, sec 0 = 1 and the gradient is 3. Since $\sec^2$ is never below 1, the gradient never drops below 3: tan 3x has no flat points anywhere. [3]