InkMaths

ANSWER BOOK

Direct impact and Newton's law of restitution

Further Mechanics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The speed of separation equals e times the speed of approach, both measured along the line of centres, with $0 \le e \le 1$. At $e = 1$ the impact is perfectly elastic and no kinetic energy is lost; at $e = 0$ the spheres do not separate at all and move off together. [2]

A2 Momentum: $3(6) = 3v_1 + 2v_2$, so $3v_1 + 2v_2 = 18$. Restitution: $v_2 - v_1 = 0.5(6) = 3$. Substituting: $3v_1 + 2(v_1 + 3) = 18$ gives $5v_1 = 12$, so $v_1 =$ 2.4 m/s and $v_2 =$ 5.4 m/s. [3]

SECTION B · Standard

B1 Before: $\frac{1}{2}(3)(6^2) =$ 54 J, the stationary sphere contributing nothing. After: $\frac{1}{2}(3)(2.4^2) + \frac{1}{2}(2)(5.4^2) = 8.64 + 29.16 =$ 37.8 J. The loss is 54 - 37.8 = 16.2 J, about 30% of the original. [4]

B2 Momentum: $mu = mv_1 + mv_2$, so $v_1 + v_2 = u$. Restitution with $e = 1$: $v_2 - v_1 = u$. Adding the two gives $2v_2 = 2u$, so $v_2 =$ u, and subtracting gives $v_1 =$ 0. The striking sphere stops dead and the struck one moves off at the original speed. Checking the energy: $\frac{1}{2}mu^2$ before and $\frac{1}{2}mu^2$ after, so nothing is lost, as $e = 1$ requires. [4]

B3 During the impact each sphere exerts a force on the other that is equal in size and opposite in direction, for the same length of time, so the impulses cancel and the total momentum cannot change. Energy has no such law: some of the kinetic energy goes into deforming the spheres, and into sound and heat, and does not come back unless the impact is perfectly elastic. [3]

SECTION C · Stretch

C1 Take the 4 kg sphere's original direction as positive, so $u_1 = 5$, $u_2 = -2$ and $v_1 = -1$. Momentum: $4(5) + 6(-2) = 8$, so $4(-1) + 6v_2 = 8$ and $v_2 =$ 2 m/s. Approach speed = 5 - (-2) = 7; separation speed = 2 - (-1) = 3. So $e = 3/7 =$ 0.429, which lies between 0 and 1 as it must. [4]