inkMaths

ANSWER BOOK

Discrete random variables and expectation

Further Statistics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The probabilities must total 1: $k(1 + 2 + 3 + 4) = 10k = 1$, so $k = 1/10$. Checking validity before anything else is what makes every later answer trustworthy. [2]

A2 $E(X) = 1(1/6) + 2(1/3) + 3(1/3) + 4(1/6) = 1/6 + 2/3 + 1 + 2/3 = 5/2$. The distribution is symmetric about 2.5, so the mean sits exactly there, and no listing was strictly needed. [3]

SECTION B · Standard

B1 $E(X^2) = 1(1/6) + 4(1/3) + 9(1/3) + 16(1/6) = 1/6 + 4/3 + 3 + 8/3 = 43/6$. Then $Var(X) = 43/6 - (5/2)^2 = 43/6 - 25/4 = 11/12 \approx 0.917$. The square goes on the mean, not on the whole expression. [4]

B2 $E(3X - 1) = 3(5/2) - 1 = 13/2$. $Var(3X - 1) = 3^2 \times 11/12 = 33/4$: the -1 shifts the centre and leaves the spread untouched, while the 3 enters the variance squared. [3]

B3 $E(X) = 0(0.7) + 1(0.2) + 5(0.1) = 0.7$, so the expected payout is 70p. A stake above 70p loses money on average; a fair game charges exactly the expected payout. Note that 70p is not a possible outcome, which is normal for a mean. [4]

SECTION C · Stretch

C1 The proposed model forces the mean and variance to be equal, which is the Poisson signature. The sample mean matches well, but the sample variance is more than twice the mean: the data are far more spread out than the model allows. The mean alone would have been misleading, which is why both moments are compared. [4]