



ANSWER BOOK

Disproof and proof by contradiction

Proof · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** Try $n = 3$: $2^3 + 1 = 9 = 3 \times 3$, which is not prime. One counter example, evaluated and factorised, finishes the job. The claim survives $n = 1$ and $n = 2$ (giving 3 and 5), which is exactly why it looks safe. [2]
- A2** Take $a = -2$ and $b = 2$. Then $a^2 = 4 = b^2$ but $a \neq b$. Squaring throws away the sign, so equal squares only force $a = \pm b$, and any pair with opposite signs is a counter example. [2]
- A3** Assume the statement is false: $\sqrt{3}$ is rational, so $\sqrt{3} = a/b$ where a and b are integers, $b \neq 0$, and the fraction is in lowest terms, meaning a and b share no common factor. The lowest-terms clause is there for a reason; the final contradiction crashes into it. [2]

SECTION B · Standard

- B1** Assume $\sqrt{3} = a/b$ in lowest terms. Squaring: $3 = a^2/b^2$, so $a^2 = 3b^2$. Then a^2 is divisible by 3, so a is too: write $a = 3c$. Substituting, $9c^2 = 3b^2$, so $b^2 = 3c^2$, and b is also divisible by 3. Now a and b share the factor 3, contradicting lowest terms. The assumption was false, so $\sqrt{3}$ is irrational. [2]
- B2** Assume such integers exist. The left side is $3(2a + 3b)$, a multiple of 3, so 1 would be a multiple of 3. But 1 is not divisible by 3: contradiction, so no such integers exist. The method amounts to spotting the common factor of the coefficients, and it is why $21a + 14b = 1$ fails the same way with a factor of 7. [3]

SECTION C · Stretch

- C1** Assume the conclusion fails: n^2 is even but n is odd, so $n = 2k + 1$. Then $n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, which is odd. That contradicts n^2 being even, so n must be even. This little fact is the hinge of the $\sqrt{2}$ proof, which is why it earns its own question. [2]
- C2** Dividing N by any prime on the list leaves remainder 1, because the product part is exactly divisible and the $+ 1$ is not: so none of 2, 3, 5, 7, 11, 13 divides N . But $N = 30031 = 59 \times 509$, so N is composite. The argument only needs some prime factor of N to be missing from the list; here 59 and 509 both are, and either one finishes the proof. [4]