



QUESTION WORKBOOK

Disproof and proof by contradiction

Proof · Year 12–13

7 questions · 20 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

A1 Disprove the statement “ $2^n + 1$ is prime for every positive integer n ”. [2]

A2 Disprove the statement “if $a^2 = b^2$ then $a = b$ ”. [2]

A3 A proof by contradiction of “ $\sqrt{3}$ is irrational” must begin with an assumption. Write down that assumption in full. [2]

SECTION B · Standard

B1 Prove by contradiction that $\sqrt{3}$ is irrational. You may use the fact that if a^2 is divisible by 3 then a is divisible by 3. [4]

B2 Prove by contradiction that there are no integers a and b with $6a + 9b = 1$. [3]

SECTION C · Stretch

C1 Prove by contradiction that if n^2 is even then n is even. [3]

C2 In the infinitely-many-primes proof, the number $N = 2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1$ is built from the primes up to 13. Show that no prime on that list divides N , and explain why the common claim “so N must be prime” is wrong, using N itself. [4]