inkMaths

ANSWER BOOK

Dynamic programming

Decision Mathematics 2 · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

A1 Any part of an optimal path is itself optimal between its own two ends. It allows the problem to be solved backwards: once the best value from every vertex onwards is known, the best value at the vertex before it is one comparison away, and no route already rejected needs reconsidering. [2]

A2 Minimum total: min(6 + 11, 10 + 5) = min(17, 15) = 15, through the second option.
Minimax: min(max(6, 11), max(10, 5)) = min(11, 10) = 10, also through the second, but chosen because its worst single stage is smaller rather than because its total is. [2]

SECTION B · Standard

B1 Stage 3: from C the value is 6, from D it is 7.
Stage 2: A gives min(4 + 6, 9 + 7) = min(10, 16) = 10 through C; B gives min(8 + 6, 2 + 7) = min(14, 9) = 9 through D.
Stage 1: S gives min(5 + 10, 3 + 9) = min(15, 12) = 12 through B.
The route is S, B, D, T of length 12. Checking all four routes gives 12, 15, 17 and 21, confirming it. [6]

B2 The two happen to agree here, which is exactly why the example is a poor test of the method. The choice was made by comparing 5 + 10 with 3 + 9, both of which use values found from the far end. Had the value at A been 5 rather than 10, S to A would have won at 10 despite the dearer first step. A greedy first move carries no guarantee about what follows it. [3]

B3 The stage says how far through the process you are: here there are three stages, one per arc travelled. The state says where you are within that stage: at stage 2 the states are A and B, at stage 3 they are C and D. Every row of the table is one (stage, state) pair, and naming both is part of the answer. [3]

SECTION C · Stretch

C1 [5]

This is a maximin problem, so the addition is replaced by taking the minimum along a route.
Stage 3: C is worth 6 and D is worth 7, as before.
Stage 2: A gives max(min(4, 6), min(9, 7)) = max(4, 7) = 7 through D; B gives max(min(8, 6), min(2, 7)) = max(6, 2) = 6 through C.
Stage 1: S gives max(min(5, 7), min(3, 6)) = max(5, 3) = 5 through A.
The route is S, A, D, T with smallest stage 5. It is a different route from the shortest one, which shows that the two objectives genuinely pull apart: S, A, D, T is the longest route of the four by total length, at 21.