inkMaths

ANSWER BOOK

Elastic potential energy

Further Mechanics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 EPE = $\lambda x^2/(2l)$. The tension grows linearly from 0 to $\lambda x/l$ as the string stretches, so the work done is the area of the triangle under the Hooke's law graph: half the base x times the height $\lambda x/l$. Using the final tension throughout would double the answer. [2]

A2 EPE = $80 \times 0.5^2/(2 \times 2) = 80 \times 0.25/4$ = 5 J. Checking by area: the tension at that extension is 20 N, and $\frac{1}{2} \times 0.5 \times 20 = 5$ J. [3]

SECTION B · Standard

B1 EPE = $20 \times 0.4^2/(2 \times 1) = 20 \times 0.16/2$ = 1.6 J. The table is smooth and horizontal, so all of it becomes kinetic energy: $\frac{1}{2}(0.2)v^2 = 1.6$ gives $v^2 = 16$ and v = 4 m/s. Beyond the natural length the string goes slack and the particle keeps that speed. [4]

B2 Elastic energy released = $39.2 \times 0.5^2/(2 \times 1)$ = 4.9 J. The particle rises 0.5 m, gaining 0.5(9.8)(0.5) = 2.45 J of potential energy. What remains is kinetic: $\frac{1}{2}(0.5)v^2 = 2.45$, so $v^2 = 9.8$ and v = 3.13 m/s. Forgetting the height change would give 4.43 m/s. [4]

B3 Using the total length rather than the extension: write l and x down separately before substituting. And measuring heights from different levels at the two instants: choose one zero level, mark it on the diagram, and use it in both totals. A third, less common, is forgetting that a string stores nothing once slack. [3]

SECTION C · Stretch

C1 Energy stored = $50 \times 0.1^2/(2 \times 0.5)$ = 0.5 J. The frictional force is 0.2 × 1 × 9.8 = 1.96 N, and all the elastic energy is eventually spent against it. So 1.96d = 0.5 and d = 0.255 m. The particle leaves the spring after 0.1 m and slides the remaining 0.155 m on its own. [4]