inkMaths

ANSWER BOOK

Estimators, standard error and confidence intervals

Further Statistics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 An estimator is unbiased if its expected value equals the parameter it estimates, so that it is right on average over repeated samples. The sample variance with divisor n - 1, $s^2 = \Sigma(x - \text{sample mean})^2/(n - 1)$, is unbiased for σ^2; dividing by n instead gives an estimate that is too small on average. [2]

A2 Standard error = $12/\sqrt{36}$ = 2. The interval is 48 ± 1.96 × 2 = 48 ± 3.92, that is (44.08, 51.92). [3]

SECTION B · Standard

B1 Using z = 2.5758: 48 ± 2.5758 × 2 = 48 ± 5.15, that is (42.85, 53.15). It is wider than the 95% interval by about 1.23 units at each end. More confidence is bought with less precision, and only a larger sample improves both at once. [4]

B2 The total width is $2 \times 1.96 \times 12/\sqrt{n}$, and setting that no more than 4 gives $\sqrt{n} \ge 2 \times 1.96 \times 12/4 = 11.76$, so $n \ge 138.3$. Since n must be a whole number, take n = 139. Halving the width again would need four times as many readings. [4]

B3 Both are unbiased: their expected values are $(\mu + \mu)/2 = \mu$ and $(\mu + 2\mu)/3 = \mu$. Their variances are $\sigma^2/2 = 0.5\sigma^2$ and $(1 + 4)\sigma^2/9$ = $0.556\sigma^2$. The simple mean has the smaller variance, so it is the more efficient: unbiasedness alone does not settle which estimator to use. [3]

SECTION C · Stretch

C1 The population mean is a fixed number, not a random one, so once the interval has been calculated it either contains μ or it does not; there is no probability left. What varies from sample to sample is the interval. The correct statement is that the method produces intervals of which 95% would contain μ over repeated sampling, so there is 95% confidence in the procedure rather than a probability attached to this one interval. [4]