inkMaths

ANSWER BOOK

Exponential functions and e

Exponentials and logarithms · Year 12

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The intercept is (0, 1), since $3^0 = 1$; at $x = 2$, $y = 9$; and the asymptote is $y = 0$, the x-axis, which the curve approaches on the left without ever touching. [2]

A2 Every exponential is positive everywhere: 5^x takes every value above zero and no others. The curve hugs the x-axis as x heads left but never reaches it, so zero is approached, not achieved. [2]

SECTION B · Standard

B1 The gradient function is $4e^{4x}$: k times the function, with $k = 4$. At $x = 0$ the gradient is $4e^0 = 4$. At $x = 0.5$ it is $4e^2 = 29.6$ to 3 significant figures, four times the height there. [3]

B2 The gradient function is $-2e^{-2x}$, negative everywhere: the curve falls at every point. At $x = 1$, $y = e^{-2} = 0.135$ to 3 significant figures. As x grows the curve decays towards the asymptote $y = 0$, steeply at first and ever more gently. [3]

B3 $P = 500 \times 3^t$: each unit step in t multiplies by the same ratio 3, which is what makes the growth exponential. After 4 hours, $P = 500 \times 81 = 40\ 500$. [3]

SECTION C · Stretch

C1 The gradient function is $5 \times 0.2e^{0.2x} = 0.2 \times 5e^{0.2x} = 0.2y$: the rate of change is proportional to the amount, with constant 0.2. At $x = 5$ the height is $5e = 13.6$ and the gradient is $0.2 \times 5e = e = 2.72$. This proportionality is the property that makes e the modelling base. [4]

C2 A gradient of exactly 1 at (0, 1) must belong to some base between 2 and 3, and that base is $e = 2.718...$. The defining property is that $y = e^x$ is its own gradient function: at every point the gradient equals the height, not just at (0, 1). [3]