inkMaths

ANSWER BOOK

First order equations and integrating factors

Differential equations · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

A1 IF = $e^{\int P\,dx}$. After multiplying through, the left side is exactly the derivative of (IF × y), so one integration solves the equation. [2]

A2 $\int \tan x\,dx = \ln(\sec x)$, so IF = $e^{\ln \sec x} = \sec x$. Log-form integrals collapse when exponentiated; leaving the factor as an unsimplified exponential wastes the marks that follow. [3]

SECTION B · Standard

B1 IF = $e^{\ln x} = x$, so $d(xy)/dx = 6x^2$. Integrating: $xy = 2x^3 + c$, and $y(1) = 3$ gives $3 = 2 + c$, $c = 1$. So $y = 2x^2 + 1/x$. Check: $dy/dx + y/x = (4x - 1/x^2) + (2x + 1/x^2) = 6x$. [5]

B2 IF = e^{-2x}: $d(e^{-2x}y)/dx = e^{3x}$. Integrating: $e^{-2x}y = e^{3x}/3 + C$, so $y = e^{5x}/3 + Ce^{2x}$. With a negative P the factor's exponent is negative too; the sign travels into the IF unchanged. [4]

B3 IF = e^{4x} gives $y = 2 + Ce^{-4x}$. Whatever the starting value, the transient Ce^{-4x} dies away and every solution settles onto the steady state $y = 2$. [3]

SECTION C · Stretch

C1 $P = 2x$, so IF = e^{x^2}. Then $d(e^{x^2}y)/dx = 4x\,e^{x^2}$, and the right side integrates by recognition: $e^{x^2}y = 2e^{x^2} + C$. So $y = 2 + Ce^{-x^2}$: a steady level 2 with a bell-shaped transient. A variable P changes nothing about the method, only the shape of the factor. [4]