inkMaths

ANSWER BOOK

Flows in networks: cuts and capacity

Decision Mathematics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 9 + 6 = 15. The arc crossing back towards the source contributes nothing: it carries flow away from the sink, so it can only reduce what gets through, never add to the ceiling the cut imposes. [2]

A2 A cut separates the source from the sink, so every unit of flow arriving at the sink must cross the cut in the forward direction at least once. The forward arcs cannot carry more than their total capacity, so the flow cannot exceed that total. [2]

SECTION B · Standard

B1 Round the source, the arcs are SA and SB: 12 + 9 = 21.
Round the sink, the arcs are CT and DT: 10 + 11 = 21.
For the third cut the forward arcs are AC and DT: 7 + 11 = 18, with no arc crossing back.
So the flow cannot exceed 18, and neither of the two obvious cuts was the binding one. [5]

B2 The forward arcs are SA and BD: 12 + 8 = 20, with no arc crossing back. It is a valid bound, but a weaker one than the 18 already found, so it adds nothing. Only the smallest cut found so far is doing any work. [3]

B3 Add a supersource joined to each factory by an arc whose capacity is that factory's output, and a supersink reached from each shop by an arc whose capacity is that shop's demand. The network then has a single source and a single sink, and every method for cuts and flows applies without modification. [3]

SECTION C · Stretch

C1 Split D into D-in and D-out. The arcs AD and BD now arrive at D-in, the arc DT now leaves D-out, and a single arc from D-in to D-out carries capacity 8. Every route through D is forced along that arc, which is exactly what the restriction means.
The cut now crosses AC and the new arc rather than DT, so its capacity is 7 + 8 = 15.
The restriction has tightened the bound, since AD and BD together could have delivered 14 to D while DT could have carried 11 away. [5]