inkMaths

ANSWER BOOK

Functions in modelling

Algebra and functions · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Equal steps added: linear. Equal factors: exponential. Repeating on a fixed cycle: trigonometric. Diagnosis comes before fitting; the behavioural signature picks the family, and only then do constants get estimated. [2]

A2 The sine runs between ±1, so h runs from 6 - 1.8 = 4.2 m up to 6 + 1.8 = 7.8 m. Centre line 6, swing 1.8: the two numbers in the model are exactly these two facts. [2]

SECTION B · Standard

B1 The sine completes a cycle when 30t reaches 360, so the period is 12 hours. High tide needs sin $(30t)°$ = 1, first at 30t = 90, so t = 3 hours. Both answers come from reading the coefficient, not from any solving. [3]

B2 At t = 0 the value is 400 + 5600 = £6000. As t grows the exponential dies away, leaving the £400 floor: scrap value. At t = 5: $V = 400 + 5600e^{-1}$ = £2460. The two constants carry the two ends of the story: start, and long run. [4]

B3 $400 + 5600e^{-0.2t} = 1200$ gives $e^{-0.2t} = 800/5600 = 1/7$. Taking logs: 0.2t = ln 7, so t = 9.73 years. The £400 must be moved across before the log is taken; logging a sum term by term is not a thing. [3]

SECTION C · Stretch

C1 At t = 12 the model is barely outside its fitted window and the growth conditions plausibly still hold: a short extrapolation. At t = 100 it predicts around 2×10^{14} bacteria, and unbounded exponential growth needs unlimited food and space, which no dish supplies. The mathematics extends for ever; the conditions that justified it do not, and the model's authority stops where the conditions do. [4]

C2 Interpolation predicts inside the range of the data the model was fitted to; extrapolation ventures beyond it. Interpolation is the safer of the two: between observed points the model is anchored on both sides, while beyond them nothing checks it. [2]