InkMaths

ANSWER BOOK

Functions, inverses and the modulus

Algebra and functions · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 fg means g first: g(2) = 4, then f(4) = 11. The other order: f(2) = 5, then g(5) = 25. The two [2]
answers differ, as composites usually do: the order genuinely matters.

A2 $fg(x) = f(x^2) = 3x^2 - 1$, and $gf(x) = (3x - 1)^2$. Feeding the inner function's output through [2]
the outer one, whole, is all that happens; the letters just track who goes first.

SECTION B · Standard

B1 Write $y = x^2 - 3$ and solve for x: $x = \sqrt{(y + 3)}$, taking the positive root because the domain [4]
restricts x to be at least 0. Swapping letters: $f^{-1}(x) = \sqrt{(x + 3)}$. The range of f is $y \geq -3$, so
the domain of f^{-1} is $x \geq -3$: the paperwork swaps roles along with the letters.

B2 $y = (2x + 1)/5$ unwinds step by step: $5y = 2x + 1$, so $x = (5y - 1)/2$. Swapping letters: $f^{-1}(x) =$ [3]
$(5x - 1)/2$. The inverse undoes the machine in reverse order: multiply by 5 first, where f
divided by 5 last.

B3 Original arm: $3x - 2 = x + 4$ gives $x = 3$, and the check $|7| = 7$ holds. Reflected arm: $-(3x -$ [4]
$2) = x + 4$ gives $2 - 3x = x + 4$, so $x = -½$, and $|-3.5| = 3.5 = -½ + 4$ holds too. Both solutions
survive: $x = -½$ and $x = 3$. Every arm's answer must be checked back, because reflected
algebra can invent crossings the graphs never make.

SECTION C · Stretch

C1 An inverse must send each output back to one input, but x^2 sends both 3 and -3 to 9: [3]
the function is not one-to-one, so no inverse exists. Restricting the domain to $x \geq 0$ (or to
$x \leq 0$) keeps one branch of the parabola, and the inverse $\sqrt{x}$ appears.

C2 The modulus measures distance from 3, so the inequality asks for points within 2 of it: [2]
< x < 5. Reading |x - a| as distance-from-a turns many modulus questions into
one-line geometry.