inkMaths

ANSWER BOOK

Further kinematics: acceleration as a function of x, t or v

Further Mechanics 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Use $v\,dv/dx$. Writing dv/dt instead leaves an equation containing v, t and x, which cannot be separated because t and x are both unknown functions of each other. Matching the form to the variable the acceleration depends on is what makes the equation separable. [2]

A2 The acceleration depends on t, so $dv/dt = 6t - 2$ and $v = 3t^2 - 2t + c$. With $v = 1$ at $t = 0$, $c = 1$, so $v = 3t^2 - 2t + 1$. At $t = 3$: $27 - 6 + 1$ = 22 m/s. [3]

SECTION B · Standard

B1 The acceleration depends on v and distance is wanted, so use $v\,dv/dx = -0.02v^2$. Cancelling one v, valid while the particle moves: $dv/v = -0.02\,dx$, so $\ln v = -0.02x + c$. With $v = 20$ at $x = 0$, $v = 20e^{(-0.02x)}$. At $x = 50$: $v = 20e^{(-1)}$ = 7.36 m/s. [4]

B2 Time is wanted and the acceleration depends on v, so $dv/dt = -0.4v$, giving $v = 15e^{(-0.4t)}$. Setting $v = 7.5$: $e^{(-0.4t)} = 0.5$, so $0.4t = \ln 2$ and t = 1.73 s. The same time would halve it again, so the speed never actually reaches zero. [4]

B3 At the terminal speed the acceleration is zero, so $2 = 0.05v^2$ and $v^2 = 40$, giving v = 6.32 m/s. No integration is needed: the terminal speed is where the driving effect and the resistance balance. [3]

SECTION C · Stretch

C1 Time is wanted and the acceleration depends on v, so $dv/dt = -(2 + 0.1v)$. Separating: $\int dv/(2 + 0.1v) = -\int dt$, so $10 \ln(2 + 0.1v) = -t + c$. At $t = 0$, $v = 20$, so $c = 10 \ln 4$. At rest $v = 0$, giving $10 \ln 2 = -t + 10 \ln 4$, so $t = 10 \ln(4/2)$ = $10 \ln 2 \approx 6.93$ s. Unlike the resistance proportional to v alone, this one does stop the particle in finite time, because of the constant term. [4]