inkMaths

ANSWER BOOK

Further loci and regions in the Argand diagram

Further Pure 2 · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

A1 Writing $z = x + iy$ and squaring both sides: $(x - 3)^2 + y^2 = 4[(x + 3)^2 + y^2]$. Expanding and collecting gives $3x^2 + 30x + 27 + 3y^2 = 0$, so $x^2 + 10x + 9 + y^2 = 0$. Completing the square: $(x + 5)^2 + y^2 = 16$, a circle of centre $(-5, 0)$ and radius 4. Ratios of distances give circles, not lines, unless the ratio is 1. [3]

A2 It is a half-line starting at the point 2 on the real axis and running up and to the right at 45° to the positive real direction, that is the part of $y = x - 2$ with x > 2. The starting point itself is excluded, because the argument of zero is undefined. An argument condition always gives a half line, never the whole line. [3]

SECTION B · Standard

B1 The argument of a quotient is the difference of the arguments, so the condition says the vector from 1 to z turns through a right angle from the vector from −1 to z. The angle subtended at z by the segment joining −1 and 1 is therefore 90°, and the angle in a semicircle puts z on the circle with that segment as diameter: $|z| = 1$. The sign of the argument selects the upper semicircle, and the two endpoints are excluded. [4]

B2 The first condition is the disc of radius 4 centred at the origin, boundary included. The second says z is at least as far from 3 as from −3, which is the half-plane $x \le 0$. The region is the left half of the disc, a semicircular area of 8π, about 25.13 square units. [4]

B3 The centre is at $-2 + i$, whose modulus is $\sqrt{5}$, about 2.236, and the radius is 3. The greatest and least distances from the origin to a point of the circle are $\sqrt{5} + 3 \approx$ 5.236 and $3 - \sqrt{5} \approx$ 0.764. Since the radius exceeds $\sqrt{5}$, the origin lies inside the circle and the least value is a genuine minimum, not zero. [3]

SECTION C · Stretch

C1 The locus is the perpendicular bisector of the segment joining 4 and 2i. That segment runs from $(4, 0)$ to $(0, 2)$ with gradient $-1/2$ and midpoint $(2, 1)$, so the bisector has gradient 2: $y = 2x - 3$. The least modulus is the perpendicular distance from the origin, $|-3|/\sqrt{(2^2 + 1^2)} = 3/\sqrt{5}$, about 1.342. [4]