



ANSWER BOOK

Game theory: play safe and stable solutions

Decision Mathematics 2 · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

- A1** The matrix is written from the row player's point of view, so if those choices are made the row player gains 5 and the column player loses 5. The game is zero-sum, so the column player is looking for small entries, not large ones. [2]
- A2** Maximin = $\max(4, 2) = 4$ and minimax = $\min(7, 4, 9) = 4$. They agree, so the game is stable with value 4, occurring where the first row meets the second column. [2]

SECTION B · Standard

- B1** Row minima are 2, 1 and 2, so the row player's maximin is 2, playing R1 or R3: both guarantee the same worst case. Column maxima are 4, 5 and 6, so the column player's minimax is 4, playing C1. Maximin 2 does not equal minimax 4, so the game is not stable and a mixed strategy is needed. All that has been established is that the value lies between 2 and 4. [5]
- B2** Row minima are 3, 4 and 1, so the maximin is 4, playing R2. Column maxima are 6, 4 and 8, so the minimax is 4, playing C2. They agree, so the game is stable with value 4, at R2 with C2. Neither player can improve by moving alone: switching row loses the row player 1 or 3, and switching column costs the column player 1 or 4. [4]
- B3** R2 beats R1 in every column, since $6 > 5$, $4 > 3$ and $8 > 7$, so R2 dominates R1. R2 also beats R3 everywhere, so R2 dominates R3: delete both rows. In the single remaining row, the column player wants the smallest of 6, 4 and 8, so C2 dominates C1 and C3 and both go. The matrix reduces to the single entry 4, which is the saddle point found before. Dominance has found the answer without any comparison of maximin with minimax. [4]

SECTION C · Stretch

- C1** Let the maximin occur in row i and the minimax in column j , and consider the entry a_{ij} [4] where they meet. It is at least the minimum of row i , since it lies in that row, so it is at least the maximin. It is at most the maximum of column j , so it is at most the minimax. Hence maximin is at most minimax, with that entry trapped between them. Equality forces that entry to equal both, so it is the smallest in its row and the largest in its column: neither player gains by moving alone, and the game is stable with that common number as its value.
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