inkMaths

ANSWER BOOK

Geometric and negative binomial distributions

Further Statistics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

inkmaths.co.uk

SECTION A · Warm-up

A1 The binomial fixes the number of trials and counts successes; the geometric fixes the number of successes at one and counts the trials needed to get it. The roles of the two quantities swap over. [2]

A2 $P(X = 4) = 0.3 \times 0.7^3 = 0.1029$: three failures then a success. The mean is $1/p = 3.33$ trials. Note the mode is 1 even though the average wait exceeds three. [3]

SECTION B · Standard

B1 Needing at least five trials means the first four all failed, so $P(X \geq 5) = 0.7^4 = 0.2401$. The whole upper tail collapses to a single power, because 'at least x' and 'the first x - 1 failed' are the same event. [3]

B2 This is negative binomial with $r = 2$ and $p = 0.25$: $P(X = 6) = {}^5C_1 \times 0.25^2 \times 0.75^4 = 5 \times 0.0625 \times 0.3164 = 0.0989$. The mean is $r/p = 8$ inspections, with variance $2(0.75)/0.0625 = 24$. [4]

B3 The final trial is pinned as a success by definition, so it is not free to move. Only the earlier $r - 1$ successes can be arranged, and only among the first $x - 1$ trials. Using ${}^xC_r$ would count arrangements in which the rth success arrives early, which describes a different event. [4]

SECTION C · Stretch

C1 With $r = 1$ the combination is ${}^{x-1}C_0 = 1$, so $P(X = x) = p(1 - p)^{x-1}$, the geometric formula. The mean r/p becomes $1/p$ and the variance $r(1 - p)/p^2$ becomes $(1 - p)/p^2$. Waiting for one success is the special case of waiting for r of them. [4]