inkMaths

ANSWER BOOK

Geometric series

Sequences and series · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Each term is three times the last, so $r = 3$, and $u_8 = 2 \times 3^7 = 4374$. Seven multiplications [2]
reach the eighth term: the off-by-one is the same one arithmetic sequences carry.

A2 The sum to infinity exists when $|r|$ < 1, so that the powers of r die away. Ratios 0.8 and [2]
-0.5 qualify; 1.2 does not, because its powers grow and the partial sums run off without settling.

SECTION B · Standard

B1 $a = 5$ and $r = 2$, so $S_{12} = 5(2^{12} - 1)/(2 - 1) = 5 \times 4095 = 20\,475$. With r > 1 the flipped [3]
dressing $a(r^n - 1)/(r - 1)$ keeps every bracket positive, which is worth the flip.

B2 The ratio is $18/24 = 0.75$, inside the window, so the infinite sum exists: $S_\infty = 24/(1 - 0.75) =$ [3]
96. Endless additions, finite total: each addition is a fixed fraction of the last, and the leftovers shrink geometrically.

B3 Write $S_n = a + ar + \ldots + ar^{n-1}$ and multiply by r: $rS_n = ar + ar^2 + \ldots + ar^n$. Subtracting, every [3]
term cancels except the first of the first line and the last of the second: $S_n(1 - r) = a - ar^n$, and dividing by $1 - r$ finishes. The cancellation is the proof; showing it explicitly is what the marks are for.

B4 After n years the balance is 2000×1.06^n, and exceeding 3000 needs 1.06^n > 1.5. Taking [4]
logs: n > log 1.5/log 1.06 = 6.96, so the first whole year past the mark is $n = 7$. The check both sides: $1.06^6 = 1.419$, still short, and $1.06^7 = 1.504$, past it.

SECTION C · Stretch

C1 $0.727272\ldots = 0.72 + 0.0072 + 0.000072 + \ldots$, a geometric series with $a = 0.72$ and $r = 0.01$. [3]
So the total is $0.72/(1 - 0.01) = 72/99 = 8/11$. Every recurring decimal is an infinite geometric series in disguise, which is why they are all fractions.