inkMaths

ANSWER BOOK

Hyperbolic functions and identities

Hyperbolic functions · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $\cosh x = (e^x + e^{-x})/2$ and $\sinh x = (e^x - e^{-x})/2$. At zero: $\cosh 0 = 1$ and $\sinh 0 = 0$, mirroring $\cos 0$ and $\sin 0$. [2]

A2 $e^{\ln 2} = 2$ and $e^{-\ln 2} = 1/2$, so $\cosh(\ln 2) = (2 + 1/2)/2 = 5/4$ and $\sinh(\ln 2) = (2 - 1/2)/2 = 3/4$. As a check, $(5/4)^2 - (3/4)^2 = 1$. [3]

SECTION B · Standard

B1 $\operatorname{arsinh} x = \ln(x + \sqrt{x^2 + 1})$, so $x = \ln(2 + \sqrt{5}) \approx 1.444$. sinh is one-to-one, so this is the only solution: no ± appears, unlike the cosh case. [4]

B2 $\operatorname{arcosh}(5/4) = \ln(5/4 + \sqrt{25/16 - 1}) = \ln(5/4 + 3/4) = \ln 2$. cosh is even, so $x = \pm\ln 2$. Every value of cosh above 1 is hit twice, once on each side of the valley. [4]

B3 Divide the whole identity by $\cosh^2 x$: $1 - \tanh^2 x = \operatorname{sech}^2 x$. The trig counterpart is $1 + \tan^2 x = \sec^2 x$; the sign flip on the $\tanh^2$ term is Osborn's rule at work, since $\tanh^2$ contains a product of two sinh terms. [3]

SECTION C · Stretch

C1 Substitute: $2 + 2\sinh^2 x - \sinh x = 2$, so $\sinh x\,(2\sinh x - 1) = 0$. Either $\sinh x = 0$, giving $x = 0$, or $\sinh x = 1/2$, giving $x = \ln(1/2 + \sqrt{5/4}) = \ln((1 + \sqrt{5})/2)$. Converting $\cosh^2$ into $\sinh^2$ first turns the equation into a quadratic in one variable. [4]