inkMaths

ANSWER BOOK

Hypothesis testing: correlation and the normal

Statistics · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The sample mean is normal with the same centre and a tighter spread: $N(\mu, \sigma^2/n)$. Averaging cancels wobble, and the variance shrinks by the factor n; the standard deviation of the mean is $\sigma/\sqrt{n}$. [2]

A2 The sample mean's standard deviation is $4/\sqrt{25} = 0.8$, so $z = (98.6 - 100)/0.8 = -1.75$. The denominator is the standard error, not the population σ: forgetting the $\sqrt{25}$ is the error this question exists to catch. [3]

SECTION B · Standard

B1 Under H_0 the sample mean is $N(100, 0.8^2)$, and $P(Z \leq -1.75) = 0.0401$. Since 0.0401 < 0.05, reject H_0: significant evidence at the 5% level that the mean mass is below 100 g. Comparison, decision, sentence in context: the mark scheme wants all three, every time. [5]

B2 Standard error $4/\sqrt{16} = 1$, so $z = (31.2 - 30)/1 = 1.2$, and $P(Z \geq 1.2) = 0.115$. Since 0.115 > 0.05, do not reject H_0: the evidence is insufficient to conclude the mean has risen. A 1.2-gram excess on 16 bags is inside the range that pure chance produces comfortably often. [4]

B3 Since 0.62 > 0.5155, reject H_0 at the 1% level: significant evidence of positive linear correlation in the population. The hypotheses are about ρ, the population coefficient; r is only its sample echo, and the table row is chosen by the sample size. [3]

SECTION C · Stretch

C1 The standard error $\sigma/\sqrt{n}$ shrinks as n grows, so the same difference is measured against a tighter yardstick and its z-value grows. A 1-gram drift on 4 bags is noise; on 400 bags the mean has no business straying that far, and the test says so. [3]