inkMaths

ANSWER BOOK

Hypothesis testing with the binomial

Statistics · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The null hypothesis H_0 states the status quo value of the parameter; the alternative H_1 states the change being tested for. The significance level is the probability threshold: evidence rarer than it, assuming H_0, counts as grounds for rejection. [2]

A2 H_0: p = 1/6, H_1: p > 1/6, where p is the probability of a six. Under H_0, $X \sim B(30, 1/6)$. The hypotheses are about p, the population parameter, never about the observed count itself. [3]

SECTION B · Standard

B1 H_0: p = 0.3, H_1: p > 0.3. Under H_0, $X \sim B(25, 0.3)$, and the observed-or-more-extreme probability is $P(X \geq 12) = 1 - P(X \leq 11) = 0.0442$. Since 0.0442 < 0.05, reject H_0: there is significant evidence at the 5% level that the spinner lands on red more often than claimed. The conclusion needs all three parts: the comparison, the decision, and the sentence in context. [5]

B2 Hunting the boundary: $P(X \geq 12) = 0.0565$, too big; $P(X \geq 13) = 0.0210$, inside. The critical region is $X \geq 13$, and the actual significance level is 0.0210, or 2.1%. Discreteness means the advertised 5% is rarely achievable exactly; the actual level is the region's true tail probability. [4]

B3 12 is outside the critical region, so do not reject H_0: the evidence is insufficient at the 5% level. That is a failure to disprove, not a proof: the data are consistent with p = 0.4, and also with plenty of nearby values. Hypothesis tests never prove the null. [3]

SECTION C · Stretch

C1 The test asks how surprising evidence this strong or stronger would be if H_0 held. Any count of 12 or more argues for H_1 at least as loudly as 12 does, so the whole tail counts. A single point probability is small even for utterly typical outcomes, which would make everything look significant. [3]