inkMaths

ANSWER BOOK

Implicit and parametric differentiation

Differentiation · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 y^3 gives $3y^2$ dy/dx: differentiate as if y were the variable, then attach dy/dx by the chain rule. The product xy needs the product rule: y + x dy/dx. These two moves are the whole of implicit differentiation. [2]

SECTION B · Standard

B1 Differentiate every term in place: 2x + 2y dy/dx = 0, so dy/dx = -x/y. At (6, 8) the gradient is -3/4. The radius to (6, 8) has gradient 8/6 = 4/3, and the two multiply to -1: the calculus agrees with the perpendicular-radius geometry. [4]

B2 Termwise: 2x + (y + x dy/dx) + 2y dy/dx = 0, the middle bracket from the product rule. Gathering: dy/dx (x + 2y) = -(2x + y), so dy/dx = -(2x + y)/(x + 2y). At (1, 2), which does lie on C since 1 + 2 + 4 = 7, the gradient is -4/5. Checking the point first costs a line and prevents a meaningless answer. [4]

B3 dy/dx = (dy/dt)/(dx/dt) = $3t^2/2t$ = 3t/2 for t ≠ 0. At t = 2 the point is (4, 8) and the gradient is 3. One clock drives both coordinates, and the gradient is one rate divided by the other; no Cartesian conversion is needed. [4]

SECTION C · Stretch

C1 dy/dx = (3 cos t)/(-3 sin t) = -cot t. At t = π/4, cot is 1, so the gradient is -1: the point (3√2/2, 3√2/2) sits at the top-right diagonal of the circle, where the tangent falls at 45°. The result matches -x/y from the implicit route. [3]

C2 Differentiating with respect to y: dx/dy = cos y, so dy/dx = 1/cos y. Since sin y = x, cos y = $\sqrt{1 - x^2}$ (positive on the principal range), giving dy/dx = $1/\sqrt{1 - x^2}$: the gradient of arcsin, found by running a rate upside down. [3]