



ANSWER BOOK

Inequalities and inequations

Further Pure 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $x - 4$ is negative when $x < 4$, and multiplying an inequality by a negative quantity [2]
reverses it, so one multiplication silently assumes a sign. The safe move is to subtract
everything to one side and combine over a common denominator before factorising.
- A2** Subtract: $(x - 1 - 2(x + 2))/(x + 2) > 0$, that is $(-x - 5)/(x + 2) > 0$. Critical values -5 [3]
and -2. Testing the three intervals: negative below -5, positive between, negative above
-2. So $-5 < x < -2$, with both endpoints excluded.

SECTION B · Standard

- B1** The right side must be positive, so $x > 0$ throughout. For $x \geq 3$ the modulus opens [4]
directly: $x - 3 < 2x$ gives $x > -3$, so all of $x \geq 3$ works. For $x < 3$ it opens with a sign
change: $3 - x < 2x$ gives $x > 1$. Combining, the solution is $x > 1$. Squaring both sides
would work here too, since both sides are positive on the region of interest.
- B2** Where $x^2 \geq 4$: $x^2 - 3x - 4 > 0$, that is $(x - 4)(x + 1) > 0$, giving $x > 4$ or $x < -1$ within [4]
that region. Where $x^2 < 4$: $4 - x^2 > 3x$, that is $x^2 + 3x - 4 < 0$, so $(x + 4)(x - 1) < 0$
and $-2 < x < 1$. Combining: $x < 1$ or $x > 4$. A sketch of the two graphs confirms the
crossings at 1 and 4.
- B3** A zero of the numerator makes the expression exactly 0, which satisfies a weak [3]
inequality, so $x = -3$ is included. A zero of the denominator leaves the expression
undefined, so $x = 2$ is excluded whatever the inequality sign. The solution is $x \leq -3$ or x
 > 2 .

SECTION C · Stretch

- C1** Subtract and combine: $(x(x - 1) - 3(x + 1))/((x + 1)(x - 1)) \leq 0$, that is $(x^2 - 4x - 3)/((x + 1)(x$
 $- 1)) \leq 0$. The numerator vanishes at $x = 2 \pm \sqrt{7}$ and the denominator at $x = \pm 1$: four
critical values. Testing the five intervals gives $-1 < x \leq 2 - \sqrt{7}$ or $1 < x \leq 2 + \sqrt{7}$. The
numerator zeros are included by the weak inequality; both denominator zeros stay out.