INKMATHS FLASHCARDS

Further Pure 2
groups, matrices and number theory

15 cards · fronts and backs

Print double-sided, flipping on the long edge, and each answer lands behind its question. Cut along the card borders.

Definitions in precise wording, the standard results the formulae booklet does not print, a check-yourself question per topic, and the slips that lose marks. The same cards run on the website with spaced-repetition scheduling, at inkmaths.co.uk/flashcards.

DEFINITION

Define: Group.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

DEFINITION

Define: Lagrange's theorem.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

STANDARD RESULT

Reduction formula:

$I_n = \int_0^{\pi/2} \sin^n x \, dx$: n I_n = ?

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STANDARD RESULT

Arc length:

cartesian, then polar = ?

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STANDARD RESULT

Surface of revolution:

about the x-axis, S = ?

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STANDARD RESULT

Eigenvalues:

2 by 2 characteristic equation = ?

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STANDARD RESULT

Diagonalisation:

M^n = ?

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DEFINITION

Define: Cayley-Hamilton theorem.

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DEFINITION

The order of any subgroup of a finite group divides the order of the group, so the order of every element divides it too.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

DEFINITION

A set with a binary operation satisfying closure, associativity, the existence of an identity, and an inverse for every element. Commutativity is not required.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

STANDARD RESULT

$\int \sqrt{(1 + ((dy)/(dx))^2)}\, dx, \int \sqrt{(r^2 + ((dr)/(d\theta))^2)}\, d\theta$

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

STANDARD RESULT

$(n - 1)\, I_{n-2}$, with $I_0 = (\pi)/(2)$, $I_1 = 1$

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

STANDARD RESULT

$\lambda^2 - (\text{trace})\lambda + \det = 0$

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

STANDARD RESULT

$2\pi \int y\, ds = 2\pi \int y \sqrt{(1 + ((dy)/(dx))^2)}\, dx$

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

DEFINITION

Every square matrix satisfies its own characteristic equation, which turns any power of it into a linear combination of lower powers and gives the inverse without cofactors.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

STANDARD RESULT

$P D^n P^{-1}$, P of eigenvectors, D of eigenvalues in step

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

DEFINITION

Define: Apollonius locus.

STANDARD RESULT

Inversion:

w = (1)/(z): image of a line missing 0?

STANDARD RESULT

Fermat's little theorem:

p prime, a not a multiple of p: $a^{p-1} \equiv$?

DEFINITION

Define: Bezout's identity.

STANDARD RESULT

Recurrence relations:

$u_{n+2} + b\,u_{n+1} + c\,u_n = 0$: u_n = ?

SPOT THE ERROR

Say what is wrong with this claim:

Permutations and combinations count the same thing, so either formula will do.

CHECK YOURSELF

A group has order 8. Explain why it cannot contain an element of order 3.

STANDARD RESULT

a circle through 0, and the map sends that circle back to a line

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

DEFINITION

The set of points whose distances from two fixed points are in a fixed ratio other than 1. It is a circle, not a line; equal distances give the perpendicular bisector instead.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

DEFINITION

The highest common factor of a and b can be written as ax + by for integers x and y, found by unwinding the Euclidean algorithm. When that factor is 1, x is the inverse of a modulo b.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

STANDARD RESULT

1 (mod p), so exponents reduce modulo p - 1

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

SPOT THE ERROR

Permutations count arrangements and combinations count selections; they differ by exactly the r! orderings of each selection. Choosing 11 from 21 gives 352716 selections but almost forty million times as many ordered line-ups.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

STANDARD RESULT

A m_1^n + B m_2^n, repeated root needs (A + Bn)m^n

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY

CHECK YOURSELF

The powers of such an element would form a subgroup of order 3, and 3 does not divide 8, contradicting Lagrange's theorem.

FURTHER PURE 2: GROUPS, MATRICES AND NUMBER THEORY