inkMaths

ANSWER BOOK

Integrating rational functions

Integration · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $(2/3)\ln|3x + 5| + c$. Differentiating back, the chain rule's 3 cancels the 1/3: the divide-by-the-inner-coefficient step is what makes the check work. [2]

A2 $(5/2)\ln|2x - 1| + c$ and $3\ln|x + 2| + c$. A constant over a linear bracket always lands on a log; the only arithmetic is the inner coefficient coming down as a divisor. [3]

SECTION B · Standard

B1 Split first: $(3x + 5)/((x + 1)(x + 2)) = 2/(x + 1) + 1/(x + 2)$, by the usual substitution of the roots. Each piece is a log: the integral is $2\ln|x + 1| + \ln|x + 2| + c$. Partial fractions exist for exactly this moment: no shelf entry covers the original fraction. [4]

B2 The top is half the derivative of the bottom: the integral is $\frac{1}{2}\ln(x^2 + 4) + c$ by the f'/f pattern. Partial fractions need the denominator to factorise into real linear brackets, and $x^2 + 4$ refuses: it has no real roots. Recognising which tool fits is what the question is really testing. [4]

B3 Split: $2/((x + 1)(x + 2)) = 2/(x + 1) - 2/(x + 2)$. Integrating: $2\ln(x + 1) - 2\ln(x + 2)$ evaluated from 0 to 1 $= 2(\ln 2 - \ln 3) - 2(\ln 1 - \ln 2) = 2\ln(4/3) = \ln(16/9)$. The log laws do the tidying; a decimal here would throw the exactness away. [4]

SECTION C · Stretch

C1 The fraction is improper: divide first. $x^2 + x + 1 = x(x + 1) + 1$, so the integrand is $x + 1/(x + 1)$. Integrating: $x^2/2 + \ln|x + 1| + c$. Division is the opening move whenever the top's degree reaches the bottom's; partial fractions only handle what remains. [3]