inkMaths

ANSWER BOOK

Integrating standard functions

Integration · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $\int \cos 2x\, dx = \frac{1}{2} \sin 2x + c$, and $\int \sin 4x\, dx = -\frac{1}{4} \cos 4x + c$. Where differentiation multiplied by k, integration divides; and sine picks up the minus on the way back. [2]

A2 $\int e^{3x}\, dx = e^{3x}/3 + c$, and $\int (1/x)\, dx = \ln|x| + c$. The modulus keeps the log defined for negative x, where $1/x$ still has perfectly good areas beneath it. [2]

SECTION B · Standard

B1 Termwise from the shelf: $e^{2x}/2 + 3 \ln|x| - (\cos 2x)/2 + c$. Three families, one line each; the only danger is the sine's returning minus sign. [3]

B2 $[3 \ln x]$ from 1 to e^2 = $3 \ln e^2 - 3 \ln 1 = 6 - 0 = 6$. The limits are e's own powers, so the logs evaluate exactly: no calculator, no rounding. [3]

B3 $[\frac{1}{3} \sin 3x]$ from 0 to $\pi/6$ = $\frac{1}{3} \sin (\pi/2) - 0 = \frac{1}{3}$. The upper limit was chosen so that 3x lands on $\pi/2$, where sine is exactly 1: radian limits and the k-factor working together. [3]

B4 No shelf entry covers $\cos^2 x$: rewrite first with the double-angle identity, $\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$. Integrating termwise: $x/2 + (\sin 2x)/4 + c$. Identities before integrals: the algebra converts an impossible integrand into two easy ones. [4]

SECTION C · Stretch

C1 $\sec^2 x$ integrates to $\tan x$, reversing the booklet's derivative fact. So the value is $\tan (\pi/3) - \tan 0 = \sqrt{3}$. An exact surd from an integral: the exact-value table reaches all the way into calculus. [3]