inkMaths

ANSWER BOOK

Integration as antidifferentiation

Integration · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Termwise by the reversed power rule: $x^3 - x^2 + 5x + c$. Differentiating the answer lands back on the integrand, a free check worth the ten seconds. [2]

A2 Differentiation destroys constants: $x^3 + 7$ and $x^3 - 2$ both differentiate to $3x^2$. Running the machine backwards therefore recovers a whole family of curves, one for each constant, and the + c holds the family's place. [2]

SECTION B · Standard

B1 Rewrite the fraction as $-3x^{-2}$: the integral is $x^6 + 3x^{-1} + c = x^6 + 3/x + c$. Raising −2 by one gives −1, and dividing by −1 flips the sign: the double negative is where this question earns its marks. [3]

B2 As powers: $2x^{1/2} + x^{-1/2}$. Raising each index by one and dividing: $(4/3)x^{3/2} + 2x^{1/2} + c$. Dividing by 3/2 multiplies by 2/3, and dividing by 1/2 doubles: fraction arithmetic is most of the work. [3]

B3 Integrating: $y = x^3 - 2x + c$, a family of curves. The point picks the member: $8 - 4 + c = 5$, so $c = 1$, and $y = x^3 - 2x + 1$. One point pins the constant; without it the answer stays a family. [4]

B4 d/dx of x^6 is $6x^5$; d/dx of $3x^{-1}$ is $-3x^{-2} = -3/x^2$; the constant dies. The result is $6x^5 - 3/x^2$, the original integrand: the Fundamental Theorem run as a checking device. [3]

SECTION C · Stretch

C1 No reversed power rule applies to the bracket as it stands: expand first. $(x^2 + 1)^2 = x^4 + 2x^2 + 1$, integrating to $x^5/5 + 2x^3/3 + x + c$. Writing $(x^2 + 1)^3/3$ is the classic wrong answer: differentiating it back produces an unwanted factor of $2x$. [3]