inkMaths

ANSWER BOOK

Integration by substitution and by parts

Integration · Year 12–13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The bracket's derivative, $2x$, sits alongside as a factor: the integrand is exactly what the chain rule leaves behind. So the integral is $(x^2 + 1)^4/4 + c$, the next power up, divided by it. [2]

A2 The top is exactly the derivative of the bottom, so the integral is $\ln(x^2 + 3) + c$ by the f'/f pattern. No modulus is needed here: $x^2 + 3$ is always positive. [3]

SECTION B · Standard

B1 $du = 2x\,dx$, so $x\,dx = du/2$, and the limits convert: $x = 0$ gives $u = 1$, $x = 2$ gives $u = 5$. The integral becomes $\tfrac{1}{2}\int_1^5 u^3\,du = \tfrac{1}{2}[u^4/4]$ from 1 to 5 $= (625 - 1)/8 = 78$. Converting the limits with the substitution saves ever returning to x. [4]

B2 Differentiate the x (it simplifies to 1) and integrate the exponential: $u = x$, $dv/dx = e^{2x}$ gives $\int xe^{2x}\,dx = (x/2)e^{2x} - \int(1/2)e^{2x}\,dx = (x/2)e^{2x} - e^{2x}/4 + c$. The trade turned a product into a bare exponential: that direction of trade is the judgement call. [4]

B3 Here the ln must be the differentiated one, since $\int \ln x\,dx$ is not yet on any shelf: $u = \ln x$, $dv/dx = x$. Then $\int x \ln x\,dx = (x^2/2)\ln x - \int(x^2/2)(1/x)\,dx = (x^2/2)\ln x - x^2/4 + c$. When a logarithm appears in a parts question, it always takes the u role. [4]

SECTION C · Stretch

C1 Parts with $u = x$, $dv/dx = \sin x$: the integral is $[-x\cos x + \sin x]$ from 0 to $\pi/2$. At the top: $0 + 1$; at the bottom: $0 + 0$. The value is exactly 1. The boundary term $-x\cos x$ vanishes at both ends, each for a different reason: $\cos(\pi/2) = 0$ above, $x = 0$ below. [3]