inkMaths

ANSWER BOOK

# Intersections, angles and distances

Further vectors · Year FM

6 questions · 20 marks

NAVY EDITION

## SECTION A · Warm-up

**A1** Parametrise: $x = 2 + \lambda$, $y = 2\lambda$, $z = 1 + 2\lambda$. Substituting: $(2 + \lambda) + 2\lambda + (1 + 2\lambda) = 13$, so $5\lambda + 3 = 13$ and $\lambda = 2$. The point is $(4, 4, 5)$, and $4 + 4 + 5 = 13$ confirms it. [3]

**A2** Line and plane: $\sin\theta = |b \cdot n|/(|b||n|)$ with the line's direction and the plane's normal. Two planes: $\cos\theta = |n_1 \cdot n_2|/(|n_1||n_2|)$ with the two normals. Sine for the first because the normal stands at right angles to the plane. [2]

## SECTION B · Standard

**B1** $b \cdot n = 1 + 2 + 2 = 5$, $|b| = 3$ and $|n| = \sqrt{3}$. So $\sin\theta = 5/(3\sqrt{3}) = 5\sqrt{3}/9$, giving $\theta = 74.2°$. Using cosine by habit would give the complement 15.8°, the angle to the normal instead of to the plane. [4]

**B2** Distance $= |2 - 2 + 6 - 3|/\sqrt{(4 + 1 + 4)} = 3/3 = 1$. Substitute the point, subtract $d$, divide by the normal's length, and keep the modulus. [3]

**B3** Normals $(1, 0, 1)$ and $(0, 1, 1)$: dot product 1, lengths $\sqrt{2}$ and $\sqrt{2}$. $\cos\theta = 1/2$, so $\theta = 60°$. The planes' angle is their normals' angle; no point of intersection is ever needed. [4]

## SECTION C · Stretch

**C1** $b \cdot n = 0$ means the line runs at right angles to the normal, so it is parallel to the plane: either inside it or clear of it. Test the anchor point $(0, 0, 5)$: $0 + 0 + 5 = 5 \neq 1$, so the line misses the plane entirely, staying parallel at a fixed distance. One substituted point settles which of the two parallel cases holds. [4]