InkMaths

ANSWER BOOK

Kinematics with variable acceleration

Mechanics · Year 12-13

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Suvat is derived assuming a constant: with a changing, its formulae are simply false. Instead $v = dx/dt$ and $a = dv/dt$, with integration running the chain backwards, constants fixed by the starting conditions. [2]

A2 $a = dv/dt = 12t^2 - 6$, so at $t = 2$, $a = 48 - 6 = 42$ m s^{-2}. One differentiation; the only care needed is not to integrate by reflex instead. [3]

SECTION B · Standard

B1 $a = 8 - 4t$ is zero at $t = 2$, where $v = 16 - 8 = 8$ m s^{-1}: the maximum, since a is positive before and negative after. Integrating, $x = 4t^2 - (2/3)t^3$ (constant zero from the origin start), so $x(3) = 36 - 18 = 18$ m. Maximum velocity comes from $a = 0$, not $v = 0$: the two conditions answer different questions. [4]

B2 Integrating: $v = 3t^2 - 12t + c$, and $v(0) = 9$ gives $c = 9$. Factorising: $v = 3(t - 1)(t - 3)$, so the particle is at rest at $t = 1$ and $t = 3$. The initial condition pins the constant before anything else can be asked. [4]

B3 $x = t^3 - 6t^2 + 9t$. The displacement is $x(3) - x(0) = 0$: back where it started. But the journey ran forward 4 m by $t = 1$, then backward 4 m by $t = 3$, so the distance travelled is 8 m. Splitting at the rest times is compulsory: integrating v straight through lets the two legs cancel. [4]

SECTION C · Stretch

C1 $x(3) = 27 - 54 + 27 = 0$, so it is at the origin, and $v(3) = 0$, so it is momentarily at rest there too: back at the start, stationary, with $a = 6$ m s^{-2} about to push it forward again. Factorising $x = t(t - 3)^2$ shows the return written into the algebra: the squared bracket is the touch. [3]